A RELATION BETWEEN HOCHSCHILD HOMOLOGY AND COHOMOLOGY FOR GORENSTEIN RINGS (ERRATUM)

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The paper [5] contains an error in the sense that Theorem 1 (the "duality theorem") is false in the generality stated. As a result the same is true for its corollaries: Proposition 3 and Corollary 6. The main conclusion, which is an affirmative answer to a question by Patrick Polo, remains valid however (see below).

That Theorem 1 is false as stated was pointed out in [2]. In general it can be seen as follows. If the conclusion of Theorem 1 is true then the Hochschild dimension (the cohomological dimension of HH^*) of the ring A is finite. So Theorem 1 must be false for every ring of infinite Hochschild dimension, and hence in particular for every ring of infinite global dimension.

Thus to save Theorem 1 we must assume that A has finite Hochschild dimension (let us say that A is *smooth* in this case). It is easy to see that in that case the proof becomes valid. The smoothness hypotheses is automatically satisfied in Proposition 2 (see [4]) but it must be added in Proposition 3 and Corollary 6.

The following lemma shows that smoothness is a reasonable condition.

Lemma. (1) *If A is commutative of finite type over the ground field k then A is smooth in the above sense if and only it is smooth in the classical sense (the "only if" part is related to the Hochschild-Kostant-Rosenberg theorem).*
(2) *If A and B are smooth then so is $A \otimes_k B$ (this is* [1, Prop. 2(2)]*).*
(3) *The following are equivalent:*
  (a) *A is smooth.*
  (b) *A^e has finite global dimension.*

Remark. If A and B have finite global dimension then this is not necessarily the case for $A \otimes_k B$. The standard counter example is given by two fields of infinite transcendence degree.

Remark. In practice A will often be a DG-algebra or an A_∞-algebra. In this case the correct notion of smoothness is that A should be a *compact object* in $D(A^e)$ ($\mathrm{Hom}_{D(A^e)}(A, -)$ should commute with direct sums). The author learnt this from a talk by Kontsevich.

It follows that in order to answer Patrick Polo's question we need to show additionally that if A is a regular minimal quotient of a semi-simple enveloping algebra then A is smooth. This follows from the result by Soergel [3] that the category of A-bimodules is equivalent to the category of quasi-coherent sheaves over a sheaf of twisted differential operators on a smooth variety. Thus A^e has finite global dimension and hence A is smooth.

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